Department of Mathematics

Pattamundai College, Pattamundai

Core 7, 3rd Semester

Paper : Partial Differential Equation and system of ordinary Differential Equations

Section–A

1. a) Write the symmetrical form of equation.

   b) What do you mean by multipliers?

   c) Write General form of Pfaffian Differential equation?

   d) What is the necessary and sufficient condition for pfaffian differential equation is integrable?

   e) Examine if the differential equation.

   $(2x^2+2xy+2xz^2+1)\,dx + dy + 2z\,dz = 0$ is integrable?

   f) By inspection solve $(y-z)\,dz + (z+x)\,dy + (y-x)\,dx = 0$?

   g) Verify that the equation $(2xyz+z^2)\,dx + x^2z\,dy + (xz+1)\,dz = 0$ is integrable?

   h) Find the primitive of the equation $a\,yz\,dx + b\,zx\,dy + c\,xy\,dz = 0$.

   i) Verify that the equation $zy\,dx - zx\,dy - y^2\,dz = 0$ is integrable?

   j) Form a partial differential equation by elimenating arbitrary constants : $z = (x^2+a)(y^2+b)$

   k) Eliminate the function 'f' from $z = e^{mx} f(x+y)$.

   l) What is complete integral of a partial differential equation?

   m) What is general integral of a partial differential equation?

   n) Define a linear partial differential equation.

   o) Define a non linear partial differential equation.

   p) Write the general from of Lagrange's Equation.

   q) Write the subsidiary equation to solve a nonlinier paartial differential equation of the Ist order.

   r) Write the general from of special type of Ist order non-liner partial differential equations of

      i) Standard from I

      ii) Standard from II

      iii) Standard from III

      iv) Standard from IV

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s) What is thegeneral form of linear partial differeniat equations with constant coefficients?

t) Classify the above equation as homogeneous equation and Non-homogeneous equation?

u) When we can say the equation is reducible?

v) When we can say the equation is irreduible?

w) If $F(a,b) \neq 0$, then $\frac{1}{F(D,D')} e^{ax+by} =$ __________

x) If $F(x,y) = e^{ax+by} \phi(x,y)$ when $\phi(x,y)$ is any function, then $\frac{1}{F(D,D)} \{ e^{ax+by} \phi(x,y)\} =$ ___________.

y) Write the General form of Laplae equation?

z) Write the mean value formula for harmonic function?

2. a) Define Green's function for laplace's equation.

b) Define Poisson's formula.

c) Write the general form of the one dimensional wave equation?

d) Define eigen functions and eigen values of vibrating string?

e) Define the general form of three dimensional wave equation?

f) State the general form of n-Dimensional wave equation?

g) State the general form of ane-dimensional heat equation.

h) In transmission line. define the following equations.

   i) Telephone equations.

   ii) Telegraph equations.

   iii) Radio equations.

Section – B

1. a) Verify the following equation are integrable?

i) $(y+z)\, dx + (z+x)\, dy + (x+y)\, dz = 0$

ii) $(y+z)\, dx - dy - dz = 0$

iii) $yzdx - azxdy - bxy\, dz = 0$

iv) $(ydx + xdy)(1-z) + xydz = 0$

v) $a^2y^2z^2dx + b^2z^2x^2dy + c^2x^2y^2dz = 0$

vi) $(x^2z-y^3)\, dx + 3xy^2dy + x^3dz = 0$

vii) $x(y^2-a^2)\,dx + y(x^2-z^2)\,dy - z(y^2-a^2)\,dz = 0$

viii) $(y^2+z^2+x^2)\,dx - 2xydy - 2xzdz = 0$

ix) $(yz+xyz)\,dx + (zx+xyz)\,dy + (xy+xyz)\,dz = 0$

x) $yz(1+x)\,dx + zx(1+y)\,dy + xy(1+z)\,dz = 0$

xi) $(y+a)^2\,dx + zdy - (y+a)\,dz = 0$

xii) $(2x+yz)\,ydx - x^2dy + (x+2z)\,y^2dz = 0$

2. Form partial differential equation by eliminating arbitrary constants :

i) $x^2+y^2+(z-c)^2 = a^2$

ii) $z = (x-a)^2 + (y-b)^2$

iii) $z = (x+a)(y+b)$

iv) $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

v) $z = ax^2 + bxy + cy^2$

vi) $2z = (ax+y)^2 + b.$

3. Eliminate the arbitrary functions from the following equations and form a partial differential equation.

i) $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$

ii) $z = x+y+f(xy)$

iii) $z = yf(x) + x\,g(y)$

iv) $z = f(x+iy) + g(x-iy)$

v) $z = xy + f(x^2+y^2)$

vi) $z = f\left(\frac{xy}{z}\right)$

4. Find the complete integral of the following :

i) $p^2-q^2 = 1$

ii) $pq = 1$

iii) $q = e^{-p/a}.$

iv) $p+q = pq$

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5. Find the complete citegral of the following :

i) $p^2+q^2 = x+y$

ii) $q = 2yp^2$

iii) $x^2p^2 = yq^2$

iv) $q = xyp^2$

6. Find the complete integral of the following :

i) $z = px + qy + c\sqrt{1+p^2+q^2}$

ii) $z = px + qy + pq$

iii) $z = px + qy - \log pq$.

iv) $z = px + qy + 3(pq)^{\frac{1}{3}}$.

7. Find the complementary function of the equation

$(D^3 + 3D^2 D' - 4D'^3)\, z = 0$?

8. Solve the following equtions.

i) $(D^2 - D'^2)\, z = 0$

ii) $(D^2D' - 4DD'^2)\, z = 0$

iii) $(D^3 - 6D^2D' + 11DD'^2 - 6D'^2)\, z = 0$

iv) $(D^3D'^2 + D^2D'^3)\, z = 0$

v) $(D^4 + D'^4 - 2D^2D'^2)\, z = 0$

vi) $(DD' + D'^2 - 3D')\, z = 0$

vii) $(D^2 - DD' - 2D)\, z = 0$

viii) $(D^2 - D'^2 - 3D + 3D')\, z = 0$

9. Find the particular integral of the equation $(D^2 - D')\, z = e^{x+y}$?

10. Find a particular integral of the equation $(D^2 - 3DD' + 2D'^2)\, z = e^{2x-y}$.

11) Find a PI of the equation $(D^2 - D')\, z = x \sin y$

Section – C

1. Solve the following equations :

i) $\dfrac{adx}{(b-c)yz} = \dfrac{bdy}{(c-a)zx} = \dfrac{cdz}{(a-b)xy}$

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ii) $\frac{dx}{x^2(y^3 - z^3)} = \frac{dy}{y^2(z^3 - x^3)} = \frac{dz}{z^2(x^3 - y^3)}$

iii) $\frac{dx}{yz} = \frac{dy}{zx} = \frac{dz}{xy}$

iv) $\frac{dx}{y^2} = \frac{dy}{y} = \frac{dz}{z}$

v) $\frac{dx}{y(x + y) + az} = \frac{dy}{x(x + y) - az} = \frac{dz}{z(x + y)}$

vi) $\frac{dx}{x(y^2 - z^2)} = \frac{dy}{-y(z^2 + x^2)} = \frac{dz}{z(x^2 + y^2)}$

2. Solve the following equations.

i) $\frac{dx}{x + z} = \frac{dy}{y} = \frac{dz}{z + y^2}$

ii) $\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{nxy}$

iii) $\frac{dx}{\cos(x + y)} = \frac{dy}{\sin(x + y)} = \frac{dz}{z}$

iv) $\frac{dz}{y + z} = \frac{dy}{z - x} = \frac{dz}{z + 5y}$

v) $\frac{xdx}{y^2z} = \frac{dy}{xz} = \frac{dz}{y^2}$

3. Solve the equation $(y^2+yz+z^2)\, dx + (z^2+zx+x^2)\, dy+(x^2+xy+y^2)dz =0$

4. Solve the equation $yzdx + (x^2y-zx)\, dy + (x^2z-xy)\, dz = 0$

5. Find the general integrals of the following partial differential equations :

i) $y^2p - xyq = x\,(z-2y)$

ii) $x\,(y-z)\,p + y\,(z-x)\,z = z\,(z-y)$

iii) $(y-zx)\,p + (x+yz)\,q = x^2+y^2$

iv) $(y+zx)\,p - (x+yz)\,q = x^2-y^2$

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v) $xzp + yzq = xy$

vi) $(3x+y-z)\,p + (x+y-z)\,q = 2(z-y)$

vii) $(z-y)\,p + (x-z)\,q = y-x$

6. Find the complete integrals of the followings.

i) $z^2 = pqxy$

ii) $px+qy = pq$

iii) $(p+q)\,(px+gy) = 1$

iv) $p^2+px+q = z$

7. Find the complete integrals of the following.

i) $p^2-q^2 = 1$

ii) $pq = 1$

iii) $q = e^{-p/a}$

iv) $p+q = pq$

v) $yp + x^2q^2 = 2x^2y$

8. Find the complete integral of each of the followings.

i) $z = p^2-q^2$

ii) $p^3+q^3 = 3pqz$

iii) $q(p^2z+q^2) = 4$

iv) $z = pq$

v) $p^2+q^2 = x+y$

vi) $q = 2yp^2$

vii) $x^2p^2 = yq^2$

viii) $(x+y)\,(p+q)^2 + (x-y)\,(p-q)^2 = 1$

ix) $\frac{p^2}{x} - \frac{q^2}{y} = \frac{1}{z}\left(\frac{1}{x}+\frac{1}{y}\right)$

x) $z = pz+qy - \log pq$

xi) $z = px+qy+c\sqrt{1+p^2+q^2}$

xii) $z = px + qy + 3\,(pq)^{\frac{1}{3}}$

9. Solve the following equations.

i) $(D^2 - a^2D'^2)\, z = x$

ii) $(D^2 - D'^2)\, z = x - y$

iii) $(D^2 - D)\, z = A \cos (lx + my)$, Where A, l, m are constants.

iv) $(D^2 + DD' - 6D'^2)\, z = y \cos x$

v) $(D^2 + 2DD' + D'^2) z = 2 \cos y - x \sin y$

vi) $(x^2D^2 - y^2D'^2 - yD' + xD)\, z = 0$

vii) $(x^2D^2 - 4xyDD' + 4y^2D'^2 + 6yD')\, z = x^3y^4$.

viii) $x^2y - y^2t = xy$.

10. Show that the only solution of the two dimensional laplace equation depending only on $r = \sqrt{x^2 + y^2}$ is $u = c\log r + k$

11. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ for $0 < x < \pi$, $0 < y < \pi$ with conditions $u(0,y) = u(\pi,y) = u(x,\pi) = 0$. $u(x,0) = \sin^2 x$.

12. Find the solution of the wave equation $\frac{1}{c^2} u_{tt} - u_{xx} - u_{yy} - u_{2z} = 0$ given that $u=0$ $u_t = x^2+xy+z^2\}$ when $t=0$

13. Obtain the solution of the radio equation $\frac{\partial^2 v}{\partial x^2} = LC \frac{\partial^2 v}{\partial t^2}$ appropriate to the case when a periodic e.m.f $v_0 \cos pt$ is applied at the end $x=0$ of the line.

14. Find a solution of $\frac{\partial v}{\partial t} = k \frac{\partial^2 v}{\partial t^2}$ having given that $V = V_0 \sin nt$ when $x=0$ for all values of t and $v = 0$ when x is very large.

15. Find a solution to the equation $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$ that satisfies the conditions $u(o,t) = 0$, $u(l,t) = 0$, $t > 0$, $u(x,0) = x$ when $0 \le x \le \frac{l}{2}$

$= l - x$ when $\frac{l}{2} < x < l$

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